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Phase dynamics and Bose-broken symmetry in atomic Bose–Einstein condensates

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Collisions lead to the collapse and revival of the interference fringes between two atomic Bose–Einstein condensates. We study the collapses and revivals of the relative phase between the condensates for two different initial states. One state invokes Bose-broken symmetry allowing us to write the wavefunction as a superposition of total number (of both condensates) states whereas the other does not.

The recent experimental realization of a weakly interacting Bose–Einstein condensate in an alkalic gas (Anderson *et al.* 1995; Bradley *et al.* 1995; Davis *et al.* 1995) has stimulated considerable theoretical work on the properties of these condensates. One question which has received much attention concerns the phase of the condensate and how it is established (Javanainen & Yoo 1996; Naraschewski *et al.* 1996; Castin & Dalibard 1997; Cirac *et al.* 1996; Wong *et al.* 1996; Barnett *et al.* 1996; Mølmer 1997). A conventional approach is to invoke Bose-broken symmetry arguments (for a comprehensive discussion of Bose-broken symmetry see, for example, Griffin (1993)) and select an arbitrary phase from an initial state which has a random phase. (It is not possible to measure the absolute phase of a condensate so that the phases we talk about are the relative phases between two condensates.) A condensate formed in the ground state of a trap will be in a number state, though we may not have knowledge of what this number is. A number state may be considered as a continuous superposition of coherent states all with different phases; spontaneous symmetry breaking then selects just one (arbitrary) phase. Recently it has been shown that a relative phase may be established between two condensates initially in number states by measurements (Javanainen & Yoo 1996; Naraschewski *et al.* 1996; Castin & Dalibard 1997; Cirac *et al.* 1996; Wong *et al.* 1996). This leads to the conclusion that the condensate can equally well be in either a coherent state or a number state. Note that whereas total atom number (of the two condensates) is specified in the number state case, an initial coherent state is a Poissonian superposition of total number states.

In this paper we consider the evolution of the visibility in the interference between two Bose–Einstein condensates in the presence of collisions. Collisions give rise to the collapse and revival of the macroscopic wavefunction of small atomic condensates composed of 10^3–10^6 atoms (Wright *et al.* 1996), which results in collapse and revival of the visibility when two condensates are interfered. The central result of this paper is that the revival time is strongly dependent on the initial state chosen for the condensate. We shall illustrate our results for two different initial states of the two condensates. In the first example, we invoke Bose-broken symmetry and consider the product of two coherent states so that superpositions of total number states are used. In the other example, we consider an entangled state of the two condensates with fixed total number N so that no assumption of Bose-broken symmetry is invoked. The evolution of the condensates under the influence of collisions is studied for each of the above initial conditions and the collapse and revival times are compared.

Our model comprises two Bose–Einstein condensates which both occupy the ground-state of their respective traps and are described by the atom annihilation (creation) operators $\hat{a}$ ($\hat{a}^\dagger$) and $\hat{b}$ ($\hat{b}^\dagger$). Atoms are released from each trap with momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, respectively, producing an interference pattern which enables a relative phase to be measured.

The intensity of the atomic field is given by

$$I(\boldsymbol{x},t) = I_0\langle[\hat{a}^\dagger(t)\mathrm{e}^{\mathrm{i}\boldsymbol{k}_1\cdot\boldsymbol{x}} + \hat{b}^\dagger(t)\mathrm{e}^{\mathrm{i}\boldsymbol{k}_2\cdot\boldsymbol{x}}][\hat{a}(t)\mathrm{e}^{-\mathrm{i}\boldsymbol{k}_1\cdot\boldsymbol{x}} + \hat{b}(t)\mathrm{e}^{-\mathrm{i}\boldsymbol{k}_2\cdot\boldsymbol{x}}]\rangle, \tag{1.1}$$

where I_0 is the single atom intensity. Atoms within each condensate collide and this may be described by the Hamiltonian

$$H = \tfrac{1}{2}\hbar\chi[(\hat{a}^\dagger\hat{a})^2 + (\hat{b}^\dagger\hat{b})^2], \tag{1.2}$$

where χ is the collision rate between the atoms within each condensate. Cross-collisions between the two condensates, described by the term $\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}$, are not included since they are dependent on the actual geometry of the physical situation. The coefficient of this term could be anywhere between zero and $\hbar\chi$ depending on the overlap between the two condensates. We consider the case where this overlap is small.

Including the time dependence of $\hat{a}$ and $\hat{b}$ due to the collisions described by equation (1.2), we get for the intensity,

$$I(\boldsymbol{x},t) = I_0\{\langle\hat{a}^\dagger\hat{a}\rangle + \langle\hat{b}^\dagger\hat{b}\rangle + \langle\hat{a}^\dagger\exp[\mathrm{i}(\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b})\chi t]\hat{b}\rangle\mathrm{e}^{-\mathrm{i}\phi(\boldsymbol{x})} + \mathrm{h.c.}\}, \tag{1.3}$$

where $\phi(\boldsymbol{x}) = (\boldsymbol{k}_2 - \boldsymbol{k}_1)\cdot\boldsymbol{x}$. The third term in the expression for $I(\boldsymbol{x},t)$ gives rise to interference fringes. However, the interference pattern will be modified in time due to the decohering effects of the collisions.

We shall now proceed to demonstrate how the interference pattern evolves for different initial states of the condensates. We will consider two classes of initial states, one which uses Bose-broken symmetry arguments and the other which does not. Invoking Bose-broken symmetry allows us to write the wavefunction as a superposition of number states. Bose symmetry is deemed to be broken in the formation of the condensate. The uncertainty in the number leads to the wavefunction possessing a definite phase via the number-phase uncertainty relation $\Delta n\Delta\phi \sim 1$. Following this line of argument, one can imagine an initial fixed number of atoms before the condensate is formed. This has indeterminate phase since it can be expressed as a continuous superposition of coherent states each possessing a particular phase. When the condensate is formed, the symmetry is broken with one of the coherent state selected, its phase becomes the phase of the condensate. In the spirit of Bose-broken symmetry we then take each condensate to be initially in a coherent state

$$|\varphi_{\mathrm{B}}\rangle = |\alpha\rangle|\beta\rangle. \tag{1.4}$$

This yields for the intensity in equation (1.3)

$$I(\boldsymbol{x},t) = I_0|\alpha|^2\{1 + \exp[2|\alpha|^2(\cos\chi t - 1)]\cos[\phi(\boldsymbol{x}) - \phi]\}, \tag{1.5}$$

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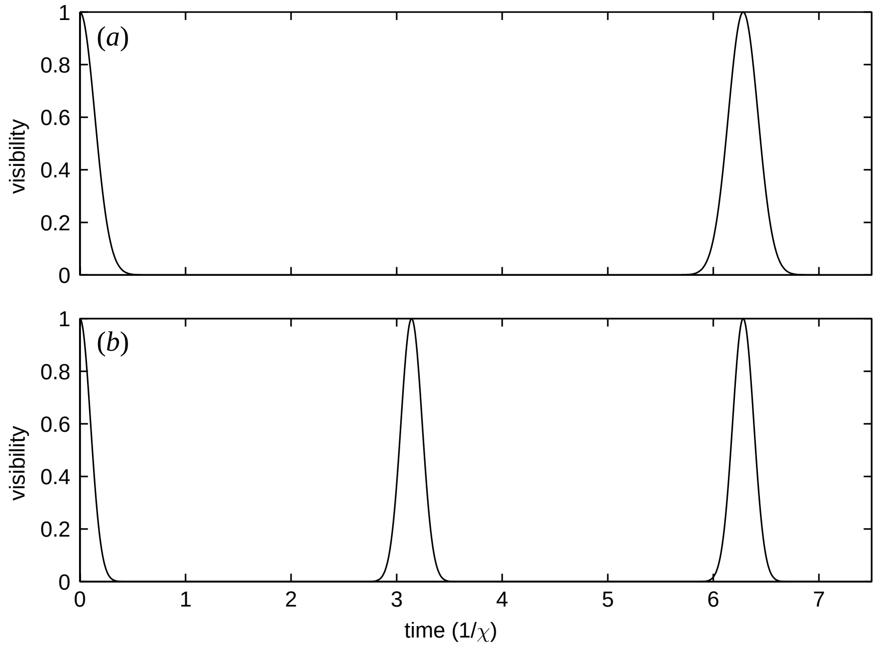

Figure 1. The visibility as a function of time for (*a*) two independent coherent states which invokes Bose-broken symmetry and (*b*) entangled coherent states which does not. We have used a total atom number of $N = 100$ corresponding to a square amplitude $|\alpha|^2 = 50$ for the two independent coherent states.

where we have set the amplitude of the two condensates to be equal to maximize the visibility of the interference pattern. The relative phase between the two condensates is defined to be ϕ, so that the relationship between the complex amplitudes is $\beta = \alpha e^{-i\phi}$. The exponential term describes the time dependence of the visibility of the interference pattern. Inside this exponential, we have a periodic function of period $2\pi/\chi$, corresponding to revival times where the visibility is 1. This visibility suffers a minimum half-way between these revivals with a value of $\exp(-4|\alpha|^2)$; it varies smoothly in time from these minima to their local maxima.

In our second class of initial states, we will not use Bose-broken symmetry, so we have an initial fixed total atom number N for the two condensates. As an example, we consider the product of two coherent states $|\alpha\rangle \otimes |\beta\rangle$ projected onto a number state basis (Mølmer 1997). This basis is truncated to size N with equal amplitudes $|\alpha| = |\beta| = \sqrt{(\frac{1}{2}N)}$. We can define a relative phase between the condensates by superposing number difference states. This entangled state is

$$|\varphi_N\rangle = 2^{-N/2} e^{iN\phi} \sqrt{N!} \sum_{k=0}^{N} \frac{e^{-ik\phi}}{\sqrt{k!\,(N-k)!}} |k\rangle \otimes |N-k\rangle, \tag{1.6}$$

where ϕ is the relative phase between the condensates. Note how each entangled number state has fixed total atom number N and number difference $2k$. Entangled states of similar form can be produced by interfering two condensates initially in number states. The intensity for this initial state is

$$I(\boldsymbol{x}, t) = I_0 \tfrac{1}{2} N \{1 + \cos^{N-1} \chi t \cos[\phi(\boldsymbol{x}) - \phi]\}. \tag{1.7}$$

The visibility of the interference pattern is $|\cos^{N-1} \chi t|$. The parity of the total atom number N, whether it is odd or even, plays a role in the revivals. When N is odd, the cosine term is raised to an even power giving a revival period of π/χ since it is never negative. When N is even, the cosine term is raised to a odd power, also giving a revival period of π/χ, but with each alternate revival occurring with the phase shifted by π radians. In both cases, the revival period is one half of the period predicted in the previous case where we used Bose-broken symmetry whatever the parity of N is.

The significant difference between the two cases is that the total number N is not fixed for the case of Bose-broken symmetry, whereas it is for the other case. The factor of two difference in the period can be explained by looking at the exponential term in equation (1.3). Inside this exponential we have the atom number difference operator $\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}$, which is quantized in units of 2 when the total number is fixed and units of 1 when it isn't fixed. Thus we have either an $\exp(\mathrm{i}2n\chi t)$ term or an $\exp(\mathrm{i}n\chi t)$ term, where n is an integer. This gives rise to the factor of two difference in the period. The visibilities of the interference patterns when Bose-broken symmetry is invoked, equation (1.5) and when it is not, equation (1.7) are shown in figures 1*a* and 1*b*.

Observations of collapses and revivals are also possible via indirect measurements schemes using light scattering as suggested by Imamoḡlu & Kennedy (1997) and Javanainen (1996), where independent condensates are coupled to a common excited state. The recent production of two overlapping condensates of ^{87}Rb in different spin states has increased the experimental feasibility of observing collapses and revivals between two atomic condensates.

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